



Oxford Cambridge and RSA

**Friday 23 June 2023 – Afternoon**

**A Level Further Mathematics B (MEI)**

**Y435/01 Extra Pure**

**Time allowed: 1 hour 15 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator



**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

**ADVICE**

- Read each question carefully before you start your answer.

## 2

- 1 A surface is defined in 3-D by  $z = 3x^3 + 6xy + y^2$ .

Determine the coordinates of any stationary points on the surface.

[7]

- 2 A sequence is defined by the recurrence relation  $4t_{n+1} - t_n = 15n + 17$  for  $n \geq 1$ , with  $t_1 = 2$ .

(a) Solve the recurrence relation to find the particular solution for  $t_n$ .

[7]

Another sequence is defined by the recurrence relation  $(n+1)u_{n+1} - u_n^2 = 2n - \frac{1}{n^2}$  for  $n \geq 1$ , with  $u_1 = 2$ .

(b) (i) Explain why the recurrence relation for  $u_n$  **cannot** be solved using standard techniques for non-homogeneous first order recurrence relations.

[1]

(ii) Verify that the particular solution to this recurrence relation is given by  $u_n = an + \frac{b}{n}$  where  $a$  and  $b$  are constants whose values are to be determined.

[5]

A third sequence is defined by  $v_n = \frac{t_n}{u_n}$  for  $n \geq 1$ .

(c) Determine  $\lim_{n \rightarrow \infty} v_n$ .

[2]

- 3 A surface,  $S$ , is defined by  $g(x, y, z) = 0$  where  $g(x, y, z) = 2x^3 - x^2y + 2xy^2 + 27z$ . The normal to  $S$  at the point  $(1, 1, -\frac{1}{9})$  and the tangent plane to  $S$  at the point  $(3, 3, -3)$  intersect at P.

Determine the position vector of P.

[8]

- 4 The set  $G$  is given by  $G = \{\mathbf{M}: \mathbf{M} \text{ is a real } 2 \times 2 \text{ matrix and } \det \mathbf{M} = 1\}$ .
- (a) Show that  $G$  forms a group under matrix multiplication,  $\times$ . You may assume that matrix multiplication is associative. [5]
- (b) The matrix  $\mathbf{A}_n$  is defined by  $\mathbf{A}_n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$  for any integer  $n$ . The set  $S$  is defined by  $S = \{\mathbf{A}_n : n \in \mathbb{Z}, n \geq 0\}$ .
- (i) Determine whether  $S$  is closed under  $\times$ . [2]
- (ii) Determine whether  $S$  is a subgroup of  $(G, \times)$ . [2]
- (c) (i) Find a subgroup of  $(G, \times)$  of order 2. [2]
- (ii) By considering the inverse of the non-identity element in any such subgroup, or otherwise, show that this is the only subgroup of  $(G, \times)$  of order 2. [2]

The set of all real  $2 \times 2$  matrices is denoted by  $H$ .

- (d) With the help of an example, explain why  $(H, \times)$  is **not** a group. [2]

- 5 The matrix  $\mathbf{P}$  is given by  $\mathbf{P} = \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}$  where  $a$  is a constant and  $a \neq 3$ .
- (a) Given that the acute angle between the directions of the eigenvectors of  $\mathbf{P}$  is  $\frac{1}{4}\pi$  radians, determine the possible values of  $a$ . [8]
- (b) You are given instead that  $\mathbf{P}$  satisfies the matrix equation  $\mathbf{I} = \mathbf{P}^2 + r\mathbf{P}$  for some rational number  $r$ .
- (i) Use the Cayley-Hamilton theorem to determine the value of  $a$  and the corresponding value of  $r$ . [4]
- (ii) Hence show that  $\mathbf{P}^4 = s\mathbf{I} + t\mathbf{P}$  where  $s$  and  $t$  are rational numbers to be determined. You should **not** calculate  $\mathbf{P}^4$ . [3]

**END OF QUESTION PAPER**

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