Oxford Cambridge and RSA

Friday 23 June 2023 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes

* 9 9 7 3 2 3 3 4 6 0 *

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

1 A surface is defined in 3-D by $z = 3x^3 + 6xy + y^2$.

Determine the coordinates of any stationary points on the surface. **[7]**

2 A sequence is defined by the recurrence relation $4t_{n+1} - t_n = 15n + 17$ for $n \geqslant 1$, with $t_1 = 2$.

(a) Solve the recurrence relation to find the particular solution for t_n. **[7]**

Another sequence is defined by the recurrence relation $(n+1)u_{n+1} - u_n^2 = 2n - \frac{1}{n^2}$ for $n \geqslant 1$, with $u_1 = 2$.

(b) **(i)** Explain why the recurrence relation for u_n **cannot** be solved using standard techniques for non-homogeneous first order recurrence relations. **[1]**

(ii) Verify that the particular solution to this recurrence relation is given by $u_n = an + \frac{b}{n}$ where a and b are constants whose values are to be determined. **[5]**

A third sequence is defined by $v_n = \frac{t_n}{u_n}$ for $n \geqslant 1$.

(c) Determine $\lim_{n \to \infty} v_n$. **[2]**

3 A surface, S, is defined by $\mathrm{g}(x, y, z) = 0$ where $\mathrm{g}(x, y, z) = 2x^3 - x^2y + 2xy^2 + 27z$. The normal to S at the point $\left(1, 1, -\frac{1}{9}\right)$ and the tangent plane to S at the point $(3, 3, -3)$ intersect at P.

Determine the position vector of P. **[8]**

4 The set G is given by $G = \{\mathbf{M}: \mathbf{M} \text{ is a real } 2 \times 2 \text{ matrix and } \det \mathbf{M} = 1\}$.

(a) Show that G forms a group under matrix multiplication, $\times$. You may assume that matrix multiplication is associative. **[5]**

(b) The matrix $\mathbf{A}_n$ is defined by $\mathbf{A}_n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$ for any integer n. The set S is defined by

$S = \{\mathbf{A}_n : n \in \mathbb{Z}, n \geqslant 0\}$.

(i) Determine whether S is closed under $\times$. **[2]**

(ii) Determine whether S is a subgroup of $(G, \times)$. **[2]**

(c) (i) Find a subgroup of $(G, \times)$ of order 2. **[2]**

(ii) By considering the inverse of the non-identity element in any such subgroup, or otherwise, show that this is the only subgroup of $(G, \times)$ of order 2. **[2]**

The set of all real 2×2 matrices is denoted by H.

(d) With the help of an example, explain why $(H, \times)$ is **not** a group. **[2]**

5 The matrix $\mathbf{P}$ is given by $\mathbf{P} = \begin{pmatrix} a & 0 \\ 2 & 3 \end{pmatrix}$ where a is a constant and $a \neq 3$.

(a) Given that the acute angle between the directions of the eigenvectors of $\mathbf{P}$ is $\frac{1}{4}\pi$ radians, determine the possible values of a. **[8]**

(b) You are given instead that $\mathbf{P}$ satisfies the matrix equation $\mathbf{I} = \mathbf{P}^2 + r\mathbf{P}$ for some rational number r.

(i) Use the Cayley-Hamilton theorem to determine the value of a and the corresponding value of r. **[4]**

(ii) Hence show that $\mathbf{P}^4 = s\mathbf{I} + t\mathbf{P}$ where s and t are rational numbers to be determined. You should **not** calculate $\mathbf{P}^4$. **[3]**

END OF QUESTION PAPER